The $N = 2$ and $N = 4$ Supersymmetric Extensions of the Lorentz- and CPT-Violating Term in Abelian Gauge Theories

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Abstract

In this work, we propose the $N = 2$ and $N = 4$ supersymmetric extensions of the Lorentz-breaking Abelian Chern-Simons term. We formulate the question of the Lorentz violation in 6 and 10 dimensions to obtain the bosonic sectors of $N = 2-$ and $N = 4-$ supersymmetries, respectively. From this, we carry out an analysis in $N = 1 - D = 4$ superspace and, in terms of $N = 1-$ superfields, we are able to write down the $N = 2$ and $N = 4$ supersymmetric versions of the Lorentz-violating action term.

1   Introduction

The formulation of physical models for the fundamental interactions in the framework of quantum field theories for point-like objects is based on a number of principles, among which Lorentz covariance and invariance under suitable gauge symmetries. However, mechanisms for the breakdown of these symmetries have been proposed and discussed in view of a number of phenomenological and experimental evidences [1, 2, 3, 4, 5]. Astrophysical observations indicate that Lorentz symmetry may be slightly violated in order to account for anisotropies. Then, one may consider a gauge theory where Lorentz symmetry breaking may be realized by

means of a term in the action. A Chern-Simons-type term may be considered that exhibits a constant background four-vector which maintains the gauge invariance but breaks down the Lorentz space-time symmetry [1].

In the context of supersymmetry (SUSY), the issue of Lorentz violation has been considered in the literature in different formulations: in ref. [6], supersymmetry is presented by introducing a suitable modification in its algebra; in ref. [7, 8], one achieves the $N = 1-$SUSY version of the Chern-Simons term by means of the conventional superspace-superfield formalism; in ref. [9], the authors adopt the idea of Lorentz breaking operators. More particularly, considering the importance of extended supersymmetries in connection with gauge theories, we propose in this work an $N = 2$ and an $N = 4$ extended supersymmetric generalization of the Lorentz-breaking Chern-Simons term in a 4-dimensional Minkowski background. We start off with the Chern-Simons term in $(1 + 5)$ and $(1 + 9)$ space-time dimensions and adopt a particular dimensional reduction method, see [10], to obtain the bosonic sector in $D = (1 + 3)$ of the $N = 2$ and $N = 4$ supersymmetric models, respectively. This is possible because in $N = 1, D = 6$- and $N = 1, D = 10$-supersymmetries, the bosonic sector has the same number of degrees of freedom as the bosonic sector of an $N = 2, D = 4$ and $N = 4, D = 4$, respectively [11]. Once the bosonic sectors are identified, we adopt an $N = 1, D = 4$-superfield formulation to write down the gauge potential and the Lorentz-violating background supermultiplets to finally set up their coupling in terms of $N = 2$ and $N = 4$ actions realized in $N = 1$-superspace. The result is projected out in component fields and we end up with the complete actions that realize the extended supersymmetric version of the Abelian Chern-Simons Lorentz-violating term.

The general organization of our work is as follows: in Section 2, we set some preliminaries for the presentation of the $N = 2$ Abelian gauge model in terms of $N = 1$ superspace and superfields. In Section 3, we focus on the task of carrying out the $N = 2$ extension of the Lorentz-violating Chern-Simons term. Next, we go one step further and reassess the discussion of Section 3 for the case of a (maximally) $N = 4$-extended gauge theory. This is the content of Section 4. Finally, in Section 5, we present our Concluding Remarks and Comments. An Appendix follows, where we collect the relevant conventions to perform the $N = 1$-superfield manipulations.

2 $N = 2$-SUSY Abelian gauge model: basic ideas

The $N = 2$ supersymmetric generalization of the Abelian gauge model can be built up by using the superfield formalism in an $N = 1$ superspace parametrized by the coordinates $(x^\mu, \theta^a, \bar{\theta}_{\dot{a}})$ [10]. The bosonic sector of the gauge action can be obtained by means of a dimensional reduction from $D = 6$ to $D = 4$ [12, 13]. The Maxwell Lagrangian in 6 dimensions is

$$\mathcal{L} = -\frac{1}{4} F_{\hat{\mu}\hat{\nu}} F^{\hat{\mu}\hat{\nu}}, \tag{2.1}$$

where $\hat{\mu} = 0, 1, 2, 3, 4, 5$. The connection $A_{\hat{\mu}}$ can be parametrized as $A_{\hat{\mu}} = (A_\mu, \varphi_1, \varphi_2)$, where $\mu = 0, 1, 2, 3$. Notice that we keep the 6 components in 4 dimensions. By adopting as an ansatz the fact that the fields have no dependence on the coordinates x^4, x^5, we obtain the $D = 4$ Lagrangian

$$\mathcal{L} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \partial_\mu\varphi\partial^\mu\varphi^*, \tag{2.2}$$

where we define $\varphi = \varphi_1 + i\varphi_2$.

This is the bosonic sector of the $N = 2$ extended supersymmetric action. The supersymmetrization of the theory above is achieved by combining superfields of the $N = 1$ superspace as supermultiplets that accommodate the ordinary fields and their superpartners. The superfields that accomplish the task of accommodating the usual fields and their respective superpartners are a scalar, Φ, and vector superfield, V, of $N = 1 - D = 4$-superspace, which play the role of the vector multiplet (Φ, V) for $N = 2 - D = 4$.

The vector superfield V in the WZ-gauge is written as:

$$V = \theta\sigma^\mu\bar{\theta}A_\mu + \theta^2\bar{\theta}\bar{\lambda} + \bar{\theta}^2\theta\lambda + \theta^2\bar{\theta}^2 D, \tag{2.3}$$

which fulfills the reality constraint, $V = V^\dagger$. The scalar superfield is written as

$$\Phi = \varphi + i\theta\sigma^\mu\bar{\theta}\partial_\mu\varphi - \frac{1}{4}\theta^2\bar{\theta}^2\Box\varphi + \sqrt{2}\theta\psi + \frac{i}{\sqrt{2}}\theta^2\partial_\mu\psi\sigma^\mu\bar{\theta} + \theta^2 f, \tag{2.4}$$

$$\bar{\Phi} = \varphi^* - i\theta\sigma^\mu\bar{\theta}\partial_\mu\varphi^* - \frac{1}{4}\theta^2\bar{\theta}^2\Box\varphi^* + \sqrt{2}\bar{\theta}\bar{\psi} + \frac{i}{\sqrt{2}}\bar{\theta}^2\theta\sigma^\mu\partial_\mu\bar{\psi} + \bar{\theta}^2 f^*, \tag{2.5}$$

which obeys the chiral condition: $\bar{D}\Phi = D\bar{\Phi} = 0$.

The $N = 1$ scalar multiplet Φ is composed by spins $(\frac{1}{2}, 0)$ and the vector multiplet encompasses the spins $(1, \frac{1}{2})$. Then, the vector hypermultiplet (Φ, V) in $N = 2$ is composed by the spins $(1, \frac{1}{2}, \frac{1}{2}, 0, 0)$ [10].

The $N = 2$-supersymmetric extension of the Maxwell action contains the bosonic gauge Lagrangian (2.2) and is written as follows:

$$\mathcal{L} = \bar{\Phi}\Phi + W^\alpha W_\alpha \delta(\bar{\theta}^2) + \bar{W}_{\dot{\alpha}}\bar{W}^{\dot{\alpha}}\delta(\theta^2), \tag{2.6}$$

where the Abelian field-strength superfield is given by:

$$W_a = -\frac{1}{4}\bar{D}^2 D_a V_{WZ}, \qquad \bar{W}_{\dot{a}} = -\frac{1}{4}D^2\bar{D}_{\dot{a}}V_{WZ}, \tag{2.7}$$

having the chirality condition: $\bar{D}W = D\bar{W} = 0$ and $DW = \bar{D}\bar{W}$.

It is clear that the Lagrangian (2.6) is invariant under $N = 1$ supersymmetry transformation and it also exhibits $N = 2$ invariance.

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3 The Lorentz-violating term in the $N = 2$ gauge model

Now, we shall look for the $N = 2$ supersymmetric version of the Chern-Simons Lorentz-breaking term. Using the fact that the bosonic sector for $N = 2$ in $D = 4$ is the same as the one for $N = 1$ in $D = 6$, we write the Chern-Simons term for $D = 6$ and perform the dimensional reducion to $D = 4$. The $D = 4$ Chern-Simons term originally proposed by [1] is

$$\mathcal{L}_{br} = \varepsilon^{\mu\nu\kappa\lambda} A_\mu \partial_\nu A_\kappa T_\lambda. \tag{3.1}$$

We adopt for $D = 6$ the Chern-Simons term in the form

$$\mathcal{L}_{br} = \varepsilon^{\hat{\mu}\hat{\nu}\hat{\kappa}\hat{\lambda}\hat{\rho}\hat{\sigma}} A_{\hat{\mu}} \partial_{\hat{\nu}} A_{\hat{\kappa}} T_{\hat{\lambda}\hat{\rho}\hat{\sigma}}. \tag{3.2}$$

The background tensor $T_{\hat{\lambda}\hat{\rho}\hat{\sigma}}$ has 20 components, and we may rewrite it as

$$T_{\hat{\lambda}\hat{\rho}\hat{\sigma}} \equiv (R_{\rho\sigma}; S_{\rho\sigma}; \partial_\mu v; \partial_\mu u),$$

where $\hat{\mu} = \mu, 4, 5$, and we consider there is no dependence of the fields on the x^4, x^5 coordinates. The fields $R_{\rho\sigma}$ and $S_{\rho\sigma}$ have 6 components each one, and the other 8 components are redefined as 2 vectors that we write as a gradient of the scalars fields v and u. Then, the number of components is reduced to 14.

As shown in the previous section, we also redefine the gauge field $A_{\hat{\mu}} \equiv (A_\mu; \varphi_1; \varphi_2)$. It is clear that $\varepsilon^{\hat{\mu}\hat{\nu}\hat{\kappa}\hat{\lambda}\hat{\rho}\hat{\sigma}} A_{\hat{\mu}} A_{\hat{\kappa}} \partial_{\hat{\nu}} T_{\hat{\lambda}\hat{\rho}\hat{\sigma}} = 0$, so we obtain, upon integration by parts, the Lagrangian as follows:

$$
\begin{aligned}
\mathcal{L}_{br} = & -\frac{1}{4}\varepsilon^{\mu\nu\kappa\lambda} F_{\mu\nu} A_\kappa \partial_\lambda v + \frac{1}{4}\varepsilon^{\mu\nu\kappa\lambda} F_{\mu\nu} \varphi_1 R_{\kappa\lambda} + \frac{1}{4}\varepsilon^{\mu\nu\kappa\lambda} F_{\mu\nu} \varphi_2 S_{\kappa\lambda} \\
& + \frac{1}{2}\varphi_1 \partial_\nu \varphi_2 \partial^\nu u - \frac{1}{2}\varphi_2 \partial_\nu \varphi_1 \partial^\nu u.
\end{aligned}
\tag{3.3}
$$

In order to carry out the supersymmetrization of the Lagrangian (3.3) by using a superspace formalism, it is advisable to define some complex field combinations that are found in the superfields we deal with. We define these bosonic fields as

$$
\begin{aligned}
B_{\mu\nu} &= S_{\mu\nu} - i\tilde{S}_{\mu\nu}, \\
H_{\mu\nu} &= R_{\mu\nu} - i\tilde{R}_{\mu\nu} \\
\varphi &= \varphi_1 + i\varphi_2, \\
r &= t + iu, \\
s &= w + iv,
\end{aligned}
\tag{3.4}
$$

Notice that we have introduced the new real scalar fields t and w that are bosonic fields but do not appear in the bosonic Lagrangian (3.3). These fields will be necessary in the supersymmetric version to maintain the balance between the bosonic and fermionic degrees of freedom present in the scalar superfields defined with complex scalar fields. Each tensor

field, $R_{\mu\nu}$ and $S_{\mu\nu}$, appears as the real part of the complex tensor field whose imaginary parts are given in terms of their dual fields, as we see in (3.4) and can be found in [14].

The superfields for the gauge sector have been defined above. Thus, we take superfields which contain the fundamental fields of the background sector plus their supersymmetric partners. These superfields are $N = 1$-multiplets that form an $N = 2$-hypermultiplet, $(S, R, \Sigma_a, \Omega_a)$. The scalar superfields that accommodate s, s^*, r and r^* are, respectively:

$$S = s + i\theta\sigma^\mu\bar{\theta}\partial_\mu s - \frac{1}{4}\theta^2\bar{\theta}^2\Box s + \sqrt{2}\theta\xi + \frac{i}{\sqrt{2}}\theta^2\partial_\mu\xi\sigma^\mu\bar{\theta} + \theta^2 h, \tag{3.5}$$

$$\bar{S} = s^* - i\theta\sigma^\mu\bar{\theta}\partial_\mu s^* - \frac{1}{4}\theta^2\bar{\theta}^2\Box s^* + \sqrt{2}\bar{\theta}\bar{\xi} + \frac{i}{\sqrt{2}}\bar{\theta}^2\theta\sigma^\mu\partial_\mu\bar{\xi} + \bar{\theta}^2 h^*, \tag{3.6}$$

$$R = r + i\theta\sigma^\mu\bar{\theta}\partial_\mu r - \frac{1}{4}\theta^2\bar{\theta}^2\Box r + \sqrt{2}\theta\zeta + \frac{i}{\sqrt{2}}\theta^2\partial_\mu\zeta\sigma^\mu\bar{\theta} + \theta^2 g, \tag{3.7}$$

$$\bar{R} = r^* - i\theta\sigma^\mu\bar{\theta}\partial_\mu r^* - \frac{1}{4}\theta^2\bar{\theta}^2\Box r^* + \sqrt{2}\bar{\theta}\bar{\zeta} + \frac{i}{\sqrt{2}}\bar{\theta}^2\theta\sigma^\mu\partial_\mu\bar{\zeta} + \bar{\theta}^2 g^*, \tag{3.8}$$

which satisfy the chiral condition: $\bar{D}S = D\bar{S} = \bar{D}R = D\bar{R} = 0$.

The spinor superfields that contain $R_{\mu\nu}, S_{\mu\nu}$ and their dual fields are written as

$$\begin{aligned}
\Sigma_a &= \tau_a + \theta^b(\varepsilon_{ba}\rho + \sigma_{ba}^{\mu\nu}B_{\mu\nu}) + \theta^2 F_a + i\theta\sigma^\mu\bar{\theta}\partial_\mu\tau_a \\
&\quad + i\theta\sigma^\mu\bar{\theta}\theta^b\partial_\mu(\varepsilon_{ba}\rho + \sigma_{ba}^{\mu\nu}B_{\mu\nu}) - \frac{1}{4}\theta^2\bar{\theta}^2\Box\tau_a,
\end{aligned} \tag{3.9}$$

$$\begin{aligned}
\bar{\Sigma}_{\dot{a}} &= \bar{\tau}_{\dot{a}} + \bar{\theta}_{\dot{b}}(-\varepsilon^{\dot{b}}_{\ \dot{a}}\rho^* - \bar{\sigma}^{\mu\nu\dot{b}}_{\quad\dot{a}}B^*_{\mu\nu}) + \bar{\theta}^2\bar{F}_{\dot{a}} - i\theta\sigma^\mu\bar{\theta}\partial_\mu\bar{\tau}_{\dot{a}} \\
&\quad - i\theta\sigma^\mu\bar{\theta}\theta_{\dot{b}}\partial_\mu(-\varepsilon^{\dot{b}}_{\ \dot{a}}\rho^* - \bar{\sigma}^{\mu\nu\dot{b}}_{\quad\dot{a}}B^*_{\mu\nu}) - \frac{1}{4}\theta^2\bar{\theta}^2\Box\bar{\tau}_{\dot{a}},
\end{aligned} \tag{3.10}$$

$$\begin{aligned}
\Omega_a &= \chi_a + \theta^b(\varepsilon_{ba}\phi + \sigma_{ba}^{\mu\nu}H_{\mu\nu}) + \theta^2 G_a + i\theta\sigma^\mu\bar{\theta}\partial_\mu\chi_a \\
&\quad + i\theta\sigma^\mu\bar{\theta}\theta^b\partial_\mu(\varepsilon_{ba}\phi + \sigma_{ba}^{\mu\nu}H_{\mu\nu}) - \frac{1}{4}\theta^2\bar{\theta}^2\Box\chi_a,
\end{aligned} \tag{3.11}$$

$$\begin{aligned}
\bar{\Omega}_{\dot{a}} &= \bar{\chi}_{\dot{a}} + \bar{\theta}_{\dot{b}}(-\varepsilon^{\dot{b}}_{\ \dot{a}}\phi^* - \bar{\sigma}^{\mu\nu\dot{b}}_{\quad\dot{a}}H^*_{\mu\nu}) + \bar{\theta}^2\bar{G}_{\dot{a}} - i\theta\sigma^\mu\bar{\theta}\partial_\mu\bar{\chi}_{\dot{a}} \\
&\quad - i\theta\sigma^\mu\bar{\theta}\theta_{\dot{b}}\partial_\mu(-\varepsilon^{\dot{b}}_{\ \dot{a}}\phi^* - \bar{\sigma}^{\mu\nu\dot{b}}_{\quad\dot{a}}H^*_{\mu\nu}) - \frac{1}{4}\theta^2\bar{\theta}^2\Box\bar{\chi}_{\dot{a}},
\end{aligned} \tag{3.12}$$

that are also chiral $\bar{D}_{\dot{b}}\Sigma_a = D_b\bar{\Sigma}_{\dot{a}} = \bar{D}_{\dot{b}}\Omega_a = D_b\bar{\Omega}_{\dot{a}} = 0$. We can notice that we have to introduce two extra background complex scalar fields, ρ and ϕ, to match the bosonic and fermionic degrees of freedom.

Now, we are interested in building up the supersymmetric action. For that, it is useful to quote the mass dimensions of the superfields previously given:

$$[\Phi] = [\bar{\Phi}] = 1, \quad [V] = 0, \quad [W_a] = [\bar{W}_{\dot{a}}] = \frac{3}{2},$$
$$[S] = [\bar{S}] = 0, \quad [\Sigma_a] = [\bar{\Sigma}_{\dot{a}}] = [\Omega_a] = [\bar{\Omega}_{\dot{a}}] = 1, \quad [R] = [\bar{R}] = 0.$$

Based on the dimensionalities, and by analysing the bosonic Lagrangian (3.3), we propose the following supersymmetric action, $\mathcal{S}_{br}$:

$$\mathcal{S}_{br} = \int d^4x d^2\theta d^2\bar{\theta} [\frac{1}{4}W^a(D_aV)S + \frac{1}{4}\bar{W}_{\dot{a}}(\bar{D}^{\dot{a}}V)\bar{S} + \frac{i}{4}\delta(\bar{\theta})W^a(\Phi + \bar{\Phi})\Sigma_a$$
$$-\frac{i}{4}\delta(\theta)\bar{W}_{\dot{a}}(\Phi + \bar{\Phi})\bar{\Sigma}^{\dot{a}} + \frac{1}{4}\delta(\bar{\theta})W^a(\Phi - \bar{\Phi})\Omega_a \tag{3.13}$$
$$-\frac{1}{4}\delta(\theta)\bar{W}_{\dot{a}}(\Phi - \bar{\Phi})\bar{\Omega}^{\dot{a}} + \frac{1}{4}\Phi\bar{\Phi}(\bar{R} + R)],$$

which is invariant under the Abelian gauge transformations:

$$\delta V = \Lambda - \bar{\Lambda} \tag{3.14}$$
$$\delta\Phi = \delta\bar{\Phi} = \delta S = \delta\bar{S} = \delta R = \delta\bar{R} = \delta\Sigma_a = \delta\bar{\Sigma}^{\dot{a}} = 0. \tag{3.15}$$

In terms of superfields, we have two sectors:

$$Gauge\ sector \quad : \quad \{V, \Phi, \bar{\Phi}\}$$
$$Background\ sector \quad : \quad \{S, \bar{S}, \Omega_a, \bar{\Omega}^{\dot{a}}, \Sigma_a, \bar{\Sigma}^{\dot{a}}, R, \bar{R}\},$$

and, in components, these two sectors have the field content below:

$$Bosonic\ gauge\ sector \quad : \quad \{A_\mu, \varphi, \varphi^*\}$$
$$Fermionic\ gauge\ sector \quad : \quad \{\lambda, \bar{\lambda}, \psi, \bar{\psi}\}$$
$$Bosonic\ background\ sector \quad : \quad \{s, s^*, R_{\mu\nu}, S_{\mu\nu}, \rho, \rho^*, \phi, \phi^*, r, r^*\}$$
$$Fermionic\ background\ sector \quad : \quad \{\xi, \bar{\xi}, \tau, \bar{\tau}, F, \bar{F}, \chi, \bar{\chi}, G, \bar{G}, \zeta, \bar{\zeta}\}.$$

We therefore observe that the action (3.13) is manifestly invariant under $N = 1$-supersymmetry. The component-field content of the $N = 2$-supersymmetry is accommodated in the $N = 1$-superfields given in equations (3.5)-(3.12). Indeed, the action (3.13) displays a larger supersymmetry, $N = 2$, realised in terms of an $N = 1$-superspace formulation.

This Lagrangian in its component-field version reads as below:

$$\mathcal{L}_{br} = +\frac{i}{8}\partial_\mu(s - s^*)\varepsilon^{\mu\kappa\lambda\nu}F_{\kappa\lambda}A_\nu - \frac{1}{8}(s + s^*)F_{\mu\nu}F^{\mu\nu} + D^2(s + s^*)$$
$$-\frac{1}{2}is\lambda\sigma^\mu\partial_\mu\bar{\lambda} - \frac{1}{2}is^*\bar{\lambda}\bar{\sigma}^\mu\partial_\mu\lambda - \frac{1}{2\sqrt{2}}\lambda\sigma^{\mu\nu}F_{\mu\nu}\xi + \frac{1}{2\sqrt{2}}\bar{\lambda}\bar{\sigma}^{\mu\nu}F_{\mu\nu}\bar{\xi}$$

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$$+\frac{1}{4}\lambda\lambda h + \frac{1}{4}\bar{\lambda}\bar{\lambda}h^* - \frac{1}{\sqrt{2}}\lambda\xi D - \frac{1}{\sqrt{2}}\bar{\lambda}\bar{\xi}D$$

$$\frac{1}{16}\varepsilon^{\mu\nu\kappa\lambda}F_{\mu\nu}(\varphi + \varphi^*)(B_{\kappa\lambda} + B_{\kappa\lambda}^*) + \frac{i}{8}F^{\mu\nu}(B_{\mu\nu} - B_{\mu\nu}^*)(\varphi + \varphi^*)$$

$$-\frac{i\sqrt{2}}{8}\tau\sigma^{\mu\nu}\psi F_{\mu\nu} - \frac{i\sqrt{2}}{8}\bar{\tau}\bar{\sigma}^{\mu\nu}\bar{\psi}F_{\mu\nu} + \frac{1}{4}\tau\sigma^\mu\partial_\mu\bar{\lambda}(\varphi + \varphi^*)$$

$$-\frac{1}{4}\bar{\tau}\bar{\sigma}^\mu\partial_\mu\lambda(\varphi + \varphi^*) + \frac{i\sqrt{2}}{4}\psi\sigma^{\mu\nu}B_{\mu\nu}\lambda + \frac{i\sqrt{2}}{4}\bar{\psi}\bar{\sigma}^{\mu\nu}B_{\mu\nu}^*\bar{\lambda}$$

$$-\frac{i}{2}D(\varphi + \varphi^*)\rho + \frac{i}{2}D^*(\varphi + \varphi^*)\rho^*$$

$$+\frac{i\sqrt{2}}{8}\lambda\psi\rho - \frac{i\sqrt{2}}{8}\bar{\lambda}\bar{\psi}\rho^* - \frac{i\sqrt{2}}{4}D\psi\tau + \frac{i\sqrt{2}}{4}D^*\bar{\psi}\bar{\tau} \qquad (3.16)$$

$$+\frac{i}{4}f\lambda\tau - \frac{i}{4}f^*\bar{\lambda}\bar{\tau} + \frac{i}{4}(\varphi + \varphi^*)\lambda F - \frac{i}{4}(\varphi + \varphi^*)\bar{\lambda}\bar{F}$$

$$-\frac{i}{16}\varepsilon^{\mu\nu\kappa\lambda}F_{\mu\nu}(\varphi - \varphi^*)(H_{\kappa\lambda} + H_{\kappa\lambda}^*) + \frac{1}{8}F^{\mu\nu}(H_{\kappa\lambda} - H_{\kappa\lambda}^*)(\varphi - \varphi^*)$$

$$-\frac{\sqrt{2}}{8}\chi\sigma^{\mu\nu}\psi F_{\mu\nu} + \frac{\sqrt{2}}{8}\bar{\chi}\bar{\sigma}^{\mu\nu}\bar{\psi}F_{\mu\nu} - \frac{i}{4}\chi\sigma^\mu\partial_\mu\bar{\lambda}(\varphi - \varphi^*)$$

$$+\frac{i}{4}\bar{\chi}\bar{\sigma}^\mu\partial_\mu\lambda(\varphi - \varphi^*) + \frac{\sqrt{2}}{4}\psi\sigma^{\mu\nu}H_{\mu\nu}\lambda - \frac{\sqrt{2}}{4}\bar{\psi}\bar{\sigma}^{\mu\nu}H_{\mu\nu}^*\bar{\lambda}$$

$$-\frac{1}{2}D(\varphi - \varphi^*)\phi + \frac{1}{2}D^*(\varphi - \varphi^*)\phi^*$$

$$+\frac{\sqrt{2}}{8}\lambda\psi\phi + \frac{\sqrt{2}}{8}\bar{\lambda}\bar{\psi}\phi^* - \frac{\sqrt{2}}{4}D\psi\chi - \frac{\sqrt{2}}{4}D^*\bar{\psi}\bar{\chi}$$

$$+\frac{1}{4}f\lambda\chi + \frac{1}{4}f^*\bar{\lambda}\bar{\chi} + \frac{1}{4}(\varphi - \varphi^*)\lambda G - \frac{1}{4}(\varphi - \varphi^*)\bar{\lambda}\bar{G}$$

$$+\frac{1}{8}\varphi\partial_\mu\varphi^*\partial^\mu(r - r^*) - \frac{1}{8}\varphi^*\partial_\mu\varphi\partial^\mu(r - r^*)$$

$$+\frac{1}{4}\partial^\mu\varphi\partial_\mu\varphi^*(r + r^*) - \frac{1}{8}\varphi\varphi^*\Box(r + r^*) - \frac{i}{4}\psi\sigma^\mu\partial_\mu\bar{\psi}(r + r^*)$$

$$+\frac{1}{4}ff^*(r + r^*) - \frac{i}{4}\psi\sigma^\mu\bar{\psi}\partial_\mu r^*$$

$$-\frac{i}{4}\varphi\zeta\sigma^\mu\partial_\mu\bar{\psi} - \frac{i}{4}\varphi^*\psi\sigma^\mu\partial_\mu\bar{\zeta} - \frac{i}{4}\psi\sigma^\mu\bar{\zeta}\partial_\mu\varphi^*$$

$$+\frac{1}{4}\varphi f^*g + \frac{1}{4}f\varphi^*g^* - \frac{1}{4}f^*\psi\zeta - \frac{1}{4}f\bar{\psi}\bar{\zeta}.$$

We point out the pieces corresponding to the bosonic action (3.3) in the complete component-field action above:

$$\frac{i}{8}\partial_\mu(s - s^*)\varepsilon^{\mu\kappa\lambda\nu}F_{\kappa\lambda}A_\nu = -\frac{1}{4}\varepsilon^{\mu\nu\kappa\lambda}F_{\mu\nu}A_\kappa\partial_\lambda v,$$

$$\frac{1}{16}\varepsilon^{\mu\nu\kappa\lambda}F_{\mu\nu}(\varphi + \varphi^*)(B_{\kappa\lambda} + B_{\kappa\lambda}^*) = \frac{1}{4}\varepsilon^{\mu\nu\kappa\lambda}F_{\mu\nu}\varphi_1 R_{\kappa\lambda},$$

$$-\frac{i}{16}\varepsilon^{\mu\nu\kappa\lambda}F_{\mu\nu}(\varphi - \varphi^*)(H_{\kappa\lambda} + H_{\kappa\lambda}^*) = \frac{1}{4}\varepsilon^{\mu\nu\kappa\lambda}F_{\mu\nu}\varphi_2 S_{\kappa\lambda},$$

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$$\frac{1}{8}\varphi\partial_\mu\varphi^*\partial^\mu(r-r^*) - \frac{1}{8}\varphi^*\partial_\mu\varphi\partial^\mu(r-r^*) \;=\; \frac{1}{2}\varphi_1\partial_\nu\varphi_2\partial^\nu u - \frac{1}{2}\varphi_2\partial_\nu\varphi_1\partial^\nu u.$$

We can notice that this Lagrangian describes the bosonic sector (3.3) and its superpartners. We find here the $N = 1$ supersymmetrization of the Chern-Simons term presented in [7], where the first term is the same as proposed by [1], considering the constant vector as the gradient of a scalar. Since the gradient vector is a constant, we have that $s = \alpha + \beta^\mu x_\mu$. We notice in our Lagrangian the presence of the bosonic real scalar fields, $t = s + s^*$ and $u = r + r^*$, and the complex scalar fields, ρ and ϕ, that do not appear in the bosonic Lagrangian (3.3). These scalar fields appear in the supersymmetric generalization in order to keep the bosonic and fermionic degrees of freedom in equal number. We can see that the bosonic fields $D, D^*, f, f^*, h, h^*, g$ and g^* play all the role of auxiliary fields. The bosonic fields $s, s^*, R_{\mu\nu}, S_{\mu\nu}, \rho, \rho^*, \phi, \phi^*, r, r^*$ and the fermionic fields $\xi, \bar{\xi}, \tau, \bar{\tau}, F, \bar{F}, \chi, \bar{\chi}, G, \bar{G}, \zeta, \bar{\zeta}$ work as background fields breaking the Lorentz invariance.

4 $N = 4$-Supersymmetric Extension of the Lorentz-Violating Action

The $N = 4$ supersymmetric generalization of the Abelian gauge model in $D = 4$ can be built up in an $N = 1$ superspace background, with coordinates $(x^\mu, \theta^a, \bar{\theta}_{\dot{a}})$ [10]. The bosonic sector of the gauge action can be obtained upon a dimensional reduction from $D = 10$ to $D = 4$. The Maxwell Lagrangian in 10 dimensions is

$$\mathcal{L} = -\frac{1}{4}F_{\hat{\mu}\hat{\nu}}F^{\hat{\mu}\hat{\nu}}, \tag{4.1}$$

where $\hat{\mu} = \mu, 4, 5, 6, 7, 8, 9$ and $\mu = 0, 1, 2, 3$. The connection $A_{\hat{\mu}}$ can be parametrised as $A_{\hat{\mu}} = (A_\mu, \varphi^I,\ I = 1, 2, 3, 4, 5, 6)$. Notice that we keep the 10 field components in 4 dimensions. Adopting the fact that the fields have no dependence on the coordinates $x^4,\ x^5,\ x^6,\ x^7, x^8,\ x^9$, we obtain the Lagrangian

$$\mathcal{L} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \partial_\mu\varphi^I\partial^\mu\varphi^I. \tag{4.2}$$

This is the bosonic sector of the $N = 4$-extended supersymmetric action. The super-symmetrization of the theory above is accomplished by defining superfields in the $N = 1$ superspace as multiplets containing the fields and their superpartners. The superfields that contain these bosonic fields and their superpartners are 6 chiral scalars, Φ^I, and vector superfield, V, of $N = 1$-superspace; put together, (Φ^I, V), they form the gauge multiplet of $N = 4$-supersymmetry.

The vector superfield V as defined in (2.3) fulfills the reality constraint, $V = V^\dagger$. The scalar superfield is written as

$$\Phi^I = \varphi^I + i\theta\sigma^\mu\bar{\theta}\partial_\mu\varphi^I - \frac{1}{4}\theta^2\bar{\theta}^2\Box\varphi^I + \sqrt{2}\theta\psi^I + \frac{i}{\sqrt{2}}\theta^2\partial_\mu\psi^I\sigma^\mu\bar{\theta} + \theta^2 f^I, \tag{4.3}$$

$$\bar{\Phi}^I = \varphi^{*I} - i\theta\sigma^\mu\bar{\theta}\partial_\mu\varphi^{*I} - \frac{1}{4}\theta^2\bar{\theta}^2\Box\varphi^{*I} + \sqrt{2}\bar{\theta}\bar{\psi}^I + \frac{i}{\sqrt{2}}\bar{\theta}^2\theta\sigma^\mu\partial_\mu\bar{\psi}^I + \bar{\theta}^2 f^{*I}, \qquad (4.4)$$

which obeys the chirality condition: $\bar{D}\Phi^I = D\bar{\Phi}^I = 0$.

The $N = 4$-supersymmetric extension of the gauge Lagrangian (4.2) is

$$\mathcal{L} = \bar{\Phi}^I\Phi^I + W^\alpha W_\alpha\delta(\bar{\theta}^2) + \bar{W}_{\dot{\alpha}}\bar{W}^{\dot{\alpha}}\delta(\theta^2), \qquad (4.5)$$

where the Abelian field-strength superfield was defined in (2.7).

It is clear that the Lagrangian (4.5) is invariant under $N = 1$-supersymmetry transformation and it has also $N = 4$ invariance.

Now, we shall look for $N = 4$ supersymmetric version of the Chern-Simons Lorentz breaking term in $D = 4$. Similarly to what we have done for $N = 2$, we shall use the fact that the bosonic sector for $N = 4$ in $D = 4$ is the same for the $N = 1$ in $D = 10$ [10]. We write the Chern-Simons term for $D = 10$ and perform its dimensional reduction to $D = 4$. We propose for $D = 10$ the Chern-Simons term in the form

$$\mathcal{L}_{br} = \varepsilon^{\hat{\mu}\hat{\nu}\hat{\kappa}\hat{\lambda}\hat{\rho}\hat{\sigma}\hat{\delta}\hat{\tau}\hat{\beta}\hat{\gamma}} A_{\hat{\mu}}\partial_{\hat{\nu}} A_{\hat{\kappa}} T_{\hat{\lambda}\hat{\rho}\hat{\sigma}\hat{\delta}\hat{\tau}\hat{\beta}\hat{\gamma}}. \qquad (4.6)$$

The background tensor $T_{\hat{\lambda}\hat{\rho}\hat{\sigma}}$ has 120 components, but we can redefine it as

$$T_{\hat{\lambda}\hat{\rho}\hat{\sigma}} \equiv (R^I_{\rho\sigma}; \partial_\mu v; \partial_\mu u^{IJ}),$$

where $\hat{\mu} = \mu, 4, 5, 6, 7, 8, 9$ is the space-time index and $I, J = 1, 2, 3, 4, 5, 6$ is an internal index. We consider that there is no dependence of the fields on the $x^4, x^5, x^6, x^7, x^8, x^9$ coordinates. Then, we have 6 anti-symmetric tensor fields $R^I_{\rho\sigma}$ with 6 components each one and 15 vectors written as gradients of 15 scalars represented by the anti-symmetric index I, J. Then, the number of components is reduced to 52.

Next, we need to redefine the gauge field as $A_{\hat{\mu}} \equiv (A_\mu; \varphi^I, I = 1, 2, 3, 4, 5, 6)$ where φ^I is real scalar fields. Observing that $\varepsilon^{\hat{\mu}\hat{\nu}\hat{\kappa}\hat{\lambda}\hat{\rho}\hat{\sigma}\hat{\delta}\hat{\tau}\hat{\beta}\hat{\gamma}} A_{\hat{\mu}} A_{\hat{\kappa}} \partial_{\hat{\nu}} T_{\hat{\lambda}\hat{\rho}\hat{\sigma}\hat{\delta}\hat{\tau}\hat{\beta}\hat{\gamma}} = 0$, we obtain, integrating by parts, the Lagrangian as follows:

$$\mathcal{L}_{br} = -\frac{1}{4}\varepsilon^{\mu\nu\kappa\lambda} F_{\mu\nu} A_\kappa \partial_\lambda v + \frac{1}{4}\varepsilon^{\mu\nu\kappa\lambda} F_{\mu\nu} \varphi^I R^I_{\kappa\lambda} + \frac{1}{2}\varphi^I \partial_\nu\varphi^J \partial^\nu u^{IJ}. \qquad (4.7)$$

As in the case of $N = 2$, we have to define some complex field combinations that can be found in superfields. We define these bosonic fields as

$$\begin{aligned}
B^I_{\mu\nu} &= R^I_{\mu\nu} - i\tilde{R}^I_{\mu\nu}, \\
\hat{\varphi}^I &= \varphi^I + i\beta^I, \\
r^{IJ} &= t^{IJ} + iu^{IJ}, \\
s &= w + iv.
\end{aligned} \qquad (4.8)$$

Notice that we have introduced the new real scalar fields β^I, t^{IJ}, w which are not present in the bosonic Lagrangian (4.7). As already pointed out, this has to be done the supersymmetric

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version to maintain the same number of degree for the matching of the bosonic and fermionic sectors of the scalar superfields defined in terms of complex scalar fields. Each tensor field, $R^I_{\mu\nu}$, appears as the real part of the complex tensor field whose imaginary parts are given in terms of their dual fields.

We now take the superfields which contain the fundamental fields of the background sector and that accommodate their supersymmetric partners. These superfields are $N = 1$-multiplets that combine to form an $N = 4$-hypermultiplet, (S, R^{IJ}, Σ^I_a). The scalar superfields that accommodate s, s^*, r^{*IJ} and r^{IJ} are, respectively, S and $\bar{S}$ (as in eqs. (3.5) and (3.6)) and R^{IJ} and $\bar{R}^{IJ}$ as cast below:

$$R^{IJ} = r^{IJ} + i\theta\sigma^\mu\bar{\theta}\partial_\mu r^{IJ} - \frac{1}{4}\theta^2\bar{\theta}^2\Box r^{IJ} + \sqrt{2}\theta\zeta^{IJ} + \frac{i}{\sqrt{2}}\theta^2\partial_\mu\zeta^{IJ}\sigma^\mu\bar{\theta} + \theta^2 g^{IJ}, \qquad (4.9)$$

$$\bar{R}^{IJ} = r^{*IJ} - i\theta\sigma^\mu\bar{\theta}\partial_\mu r^{*IJ} - \frac{1}{4}\theta^2\bar{\theta}^2\Box r^{*IJ} + \sqrt{2}\bar{\theta}\bar{\zeta}^{IJ} + \frac{i}{\sqrt{2}}\bar{\theta}^2\theta\sigma^\mu\partial_\mu\bar{\zeta}^{IJ} + \bar{\theta}^2 g^{*IJ}, \qquad (4.10)$$

which satisfy the chiral condition: $\bar{D}S = D\bar{S} = \bar{D}R^{IJ} = D\bar{R}^{IJ} = 0$.

The spinor superfields that contain $R^I_{\mu\nu}$ and their respective dual fields are written as

$$\begin{aligned}\Sigma^I_a &= \tau^I_a + \theta^b(\varepsilon_{ba}\rho^I + \sigma^{\mu\nu}_{ba}B^I_{\mu\nu}) + \theta^2 F^I_a + i\theta\sigma^\mu\bar{\theta}\partial_\mu\tau^I_a \\ &\quad + i\theta\sigma^\mu\bar{\theta}\theta^b\partial_\mu(\varepsilon_{ba}\rho^I + \sigma^{\mu\nu}_{ba}B^I_{\mu\nu}) - \frac{1}{4}\theta^2\bar{\theta}^2\Box\tau^I_a,\end{aligned} \qquad (4.11)$$

$$\begin{aligned}\bar{\Sigma}^I_{\dot{a}} &= \bar{\tau}^I_{\dot{a}} + \bar{\theta}_{\dot{b}}(-\varepsilon^{\dot{b}}_{\ \dot{a}}\rho^{*I} - \bar{\sigma}^{\mu\nu\dot{b}}_{\ \ \dot{a}}B^{*I}_{\mu\nu}) + \bar{\theta}^2\bar{F}^I_{\dot{a}} - i\theta\sigma^\mu\bar{\theta}\partial_\mu\bar{\tau}^I_{\dot{a}} \\ &\quad - i\theta\sigma^\mu\bar{\theta}\bar{\theta}_{\dot{b}}\partial_\mu(-\varepsilon^{\dot{b}}_{\ \dot{a}}\rho^{*I} - \bar{\sigma}^{\mu\nu\dot{b}}_{\ \ \dot{a}}B^{*I}_{\mu\nu}) - \frac{1}{4}\theta^2\bar{\theta}^2\Box\bar{\tau}^I_{\dot{a}},\end{aligned} \qquad (4.12)$$

that are also chiral $\bar{D}_{\dot{b}}\Sigma^I_a = D_b\bar{\Sigma}^I_{\dot{a}} = 0$. We can notice that in spinor superfields we have to introduce six extra background complex scalar fields, ρ^I, to match the bosonic and fermionic degrees of freedom.

Based on dimensional analysis arguments for the bosonic sector, as it has been done for the $N = 2$ case, and noticing that some superfields now have internal symmetry index, we propose the following $N = 4$ supersymmetric action:

$$\begin{aligned}\mathcal{S}_{br} &= \int d^4x d^2\theta d^2\bar{\theta}[\frac{1}{4}W^a(D_aV)S + \frac{1}{4}\bar{W}_{\dot{a}}(\bar{D}^{\dot{a}}V)\bar{S} + \frac{i}{4}\delta(\bar{\theta})W^a(\Phi^I + \bar{\Phi}^I)\Sigma^I_a \\ &\quad - \frac{i}{4}\delta(\theta)\bar{W}_{\dot{a}}(\Phi^I + \bar{\Phi}^I)\bar{\Sigma}^{\dot{a}I} + \frac{1}{4}\Phi^I\bar{\Phi}^J(R^{IJ} - \bar{R}^{IJ})],\end{aligned} \qquad (4.13)$$

which is invariant under gauge transformations:

$$\begin{aligned}\delta V &= \Lambda - \bar{\Lambda} \qquad &(4.14) \\ \delta\Phi^I &= \delta\bar{\Phi}^I = \delta S = \delta\bar{S} = \delta R^{IJ} = \delta\bar{R}^{IJ} = \delta\Sigma^I_a = \delta\bar{\Sigma}^{\dot{a}I} = 0. \qquad &(4.15)\end{aligned}$$

In terms of superfields, we have two sectors:

$$
\begin{aligned}
Gauge\ Sector &: \quad \{V, \Phi^I\} \\
Background\ Sector &: \quad \{\Sigma_a^I, \bar{\Sigma}^{\dot{a}I}, S, \bar{S}, R^{IJ}, \bar{R}^{IJ}\},
\end{aligned}
$$

and, in components these two sectors encompass the fields cast below:

$$
\begin{aligned}
Bosonic\ gauge\ Sector &: \quad \{A_\mu, \varphi^I, \varphi^{*I}\} \\
Fermionic\ gauge\ Sector &: \quad \{\lambda, \bar{\lambda}, \psi^I, \bar{\psi}^I\} \\
Bosonic\ background\ Sector &: \quad \{s, s^*, R_{\mu\nu}^I, \rho^I, \rho^{*I}, r^{IJ}, r^{*IJ}\} \\
Fermionic\ background\ Sector &: \quad \{\xi, \bar{\xi}, \tau^I, \bar{\tau}^I, F^I, \bar{F}^I, \zeta^{IJ}, \bar{\zeta}^{IJ}\}.
\end{aligned}
$$

We can observe that the action (4.13) is invariant under $N = 1$-supersymmetry and there is a larger symmetry, the $N = 4$-supersymmetry as well.

This $N = 4$ Lagrangian in its component-field version reads as follows:

$$
\begin{aligned}
\mathcal{L}_{br} = {}& +\frac{i}{8}\partial_\mu(s - s^*)\varepsilon^{\mu\kappa\lambda\nu}F_{\kappa\lambda}A_\nu - \frac{1}{8}(s + s^*)F_{\mu\nu}F^{\mu\nu} + D^2(s + s^*) \\
& -\frac{1}{2}is\lambda\sigma^\mu\partial_\mu\bar{\lambda} - \frac{1}{2}is^*\bar{\lambda}\bar{\sigma}^\mu\partial_\mu\lambda - \frac{1}{2\sqrt{2}}\lambda\sigma^{\mu\nu}F_{\mu\nu}\xi + \frac{1}{2\sqrt{2}}\bar{\lambda}\bar{\sigma}^{\mu\nu}F_{\mu\nu}\bar{\xi} \\
& +\frac{1}{4}\lambda\lambda h + \frac{1}{4}\bar{\lambda}\bar{\lambda}h^* - \frac{1}{\sqrt{2}}\lambda\xi D - \frac{1}{\sqrt{2}}\bar{\lambda}\bar{\xi}D \\
& \frac{1}{16}\varepsilon^{\mu\nu\kappa\lambda}F_{\mu\nu}(\varphi^I + \varphi^{*I})(B_{\kappa\lambda}^I + B_{\kappa\lambda}^{*I}) + \frac{i}{8}F^{\mu\nu}(B_{\mu\nu}^I - B_{\mu\nu}^{*I})(\varphi^I + \varphi^{*I}) \\
& -\frac{i\sqrt{2}}{8}\tau^I\sigma^{\mu\nu}\psi^I F_{\mu\nu} - \frac{i\sqrt{2}}{8}\bar{\tau}^I\bar{\sigma}^{\mu\nu}\bar{\psi}^I F_{\mu\nu} + \frac{1}{4}\tau^I\sigma^\mu\partial_\mu\bar{\lambda}(\varphi^I + \varphi^{*I}) \\
& -\frac{1}{4}\bar{\tau}^I\bar{\sigma}^\mu\partial_\mu\lambda(\varphi^I + \varphi^{*I}) + \frac{i\sqrt{2}}{4}\psi^I\sigma^{\mu\nu}B_{\mu\nu}^I\lambda + \frac{i\sqrt{2}}{4}\bar{\psi}^I\bar{\sigma}^{\mu\nu}B_{\mu\nu}^*\bar{\lambda}^I \\
& -\frac{i}{2}D(\varphi^I + \varphi^{*I})\rho^I + \frac{i}{2}D^*(\varphi^I + \varphi^{*I})\rho^{*I} \\
& +\frac{i\sqrt{2}}{8}\lambda\psi^I\rho^I - \frac{i\sqrt{2}}{8}\bar{\lambda}\bar{\psi}^I\rho^{*I} - \frac{i\sqrt{2}}{4}D\psi^I\tau^I + \frac{i\sqrt{2}}{4}D^*\bar{\psi}^I\bar{\tau}^I \\
& +\frac{i}{4}f^I\lambda\tau^I - \frac{i}{4}f^{*I}\bar{\lambda}\bar{\tau}^I + \frac{i}{4}(\varphi^I + \varphi^{*I})\lambda F^I - \frac{i}{4}(\varphi^I + \varphi^{*I})\bar{\lambda}\bar{F}^I \\
& +\frac{1}{8}\varphi^I\partial_\mu\varphi^{*J}\partial^\mu(r^{IJ} + r^{*IJ}) - \frac{1}{8}\varphi^{*J}\partial_\mu\varphi^I\partial^\mu(r^{IJ} + r^{*IJ}) \\
& +\frac{1}{4}\partial^\mu\varphi^I\partial_\mu\varphi^{*J}(r^{IJ} - r^{*IJ}) - \frac{1}{8}\varphi^I\varphi^{*J}\Box(r^{IJ} - r^{*IJ}) - \frac{i}{4}\psi^I\sigma^\mu\partial_\mu\bar{\psi}^J(r^{IJ} - r^{*IJ}) \\
& +\frac{1}{4}f^I f^{*J}(r^{IJ} - r^{*IJ}) + \frac{i}{4}\psi^I\sigma^\mu\bar{\psi}^J\partial_\mu r^{*IJ} \\
& -\frac{i}{4}\varphi^I\zeta^{IJ}\sigma^\mu\partial_\mu\bar{\psi}^J + \frac{i}{4}\varphi^{*J}\psi^I\sigma^\mu\partial_\mu\bar{\zeta}^{IJ} + \frac{i}{4}\psi^I\sigma^\mu\bar{\zeta}^{IJ}\partial_\mu\varphi^{*J} \\
& +\frac{1}{4}\varphi^I f^{*J}g^{IJ} - \frac{1}{4}f^I\varphi^{*J}g^{*IJ} - \frac{1}{4}f^{*J}\psi^I\zeta^{IJ} + \frac{1}{4}f^I\bar{\psi}^J\bar{\zeta}^{IJ}.
\end{aligned}
$$

(4.16)

We can ascertain the presence of the bosonic sector (4.7) by means of the terms below:

$$\frac{i}{8}\partial_\mu(s - s^*)\varepsilon^{\mu\kappa\lambda\nu}F_{\kappa\lambda}A_\nu = -\frac{1}{4}\varepsilon^{\mu\nu\kappa\lambda}F_{\mu\nu}A_\kappa\partial_\lambda v,$$

$$\frac{1}{16}\varepsilon^{\mu\nu\kappa\lambda}F_{\mu\nu}(\varphi^I + \varphi^{*I})(B^I_{\kappa\lambda} + B^{*I}_{\kappa\lambda}) = \frac{1}{4}\varepsilon^{\mu\nu\kappa\lambda}F_{\mu\nu}\varphi^I R^I_{\kappa\lambda},$$

$$\frac{1}{8}\varphi^I\partial_\mu\varphi^{*J}\partial^\mu(r^{IJ} + r^{*IJ}) - \frac{1}{8}\varphi^{*J}\partial_\mu\varphi^I\partial^\mu(r^{IJ} + r^{*IJ}) = \frac{1}{2}(\varphi^I\partial_\nu\varphi^J + \beta^I\partial_\nu\beta^J)\partial^\nu u^{IJ}.$$

We can notice that this Lagrangian fairly accomodates the $N = 4$ bosonic sector (4.7). We re-obtain here the $N = 1$ and $N = 2$ supersymmetrisation of the Chern-Simons term presented in ref.[7] and in (3.16), respectively. We notice that $N = 4$ Lagrangian is similar to $N = 2$ but now existing an internal index in same fields. The fields β^I, t, u^{IJ} and ρ^{IJ}, that do not appear in the bosonic Lagrangian (4.7), were introduced in order to keep the bosonic and fermionic degrees of freedom in equal number. We can see that the bosonic fields $D, D^*, f^I, f^{*I}, h, h^*, g^{IJ}$ and g^{*IJ} works as auxiliary fields. The bosonic fields $s, s^*, R^I_{\mu\nu}, \rho^I, \rho^{*I}, r^{IJ}, r^{*IJ}$ and the fermionic fields $\xi, \bar{\xi}, \tau^I, \bar{\tau}^I, F^I, \bar{F}^I, \zeta^{IJ}, \bar{\zeta}^{IJ}$ work as background fields breaking the Lorentz invariance.

5 Concluding Remarks and Comments

In the important context of studying the gauge invariant Lorentz-violating term formulated as a Chern-Simons action term , we propose here its $N = 2$ and $N = 4$ supersymmetric versions. This programme could be done in a simple way with the help of a dimensional reduction method; here, we have chosen the method à la Scherk, but it would also be interesting to contemplate other possibilities, such as the procedures à la Legendre or à la Kaluza-Klein. With our reduction scheme, we could treat the extended supersymmetric version in terms of simple $N = 1$ superspace to supersymmetrize the Chern-Simons like term, as proposed by Jackiw, written in terms of a constant background vector here parametrized as the gradient of the scalar function $\alpha + \beta_\mu x^\mu$, where α and β^μ are constants.

Another interesting point we should consider is the possibility, once we have now the full set of SUSY partners of the Lorentz-breaking vector, to express the central charges of the extended models whenever topologically non-trivial configurations are taken into account. This would allow us to impose bounds on the central charges in terms of the phenomenological constraints imposed on the vector responsible for the Lorentz covariance breakdown.

6 Appendix

The σ^μ and $\bar{\sigma}^\mu$ matrices are defined as

$$\sigma^\mu = (1, \sigma^i), \quad \bar{\sigma}^\mu = (1, -\sigma^i),$$

where σ^i are the Pauli matrices.

The $SO(3,1)$ generators are represented by the matrices

$$\sigma^{\mu\nu} = \frac{i}{4}(\sigma^\mu \bar{\sigma}^\nu - \sigma^\nu \bar{\sigma}^\mu).$$

A useful relation envolving σ matrices is

$$\sigma^\mu_{a\dot{a}} \bar{\sigma}^{\nu\dot{a}b} \sigma^\kappa_{b\dot{b}} \bar{\sigma}^{\lambda\dot{b}a} = 2(\eta^{\mu\nu}\eta^{\kappa\lambda} - \eta^{\mu\kappa}\eta^{\nu\lambda} + \eta^{\mu\lambda}\eta^{\nu\kappa} - i\varepsilon^{\mu\nu\kappa\lambda}),$$

where $\varepsilon^{0123} = -\varepsilon_{0123} = 1$.

The Grassmmanian coordinates θ and $\bar{\theta}$ have their indices lowered and raised as

$$\theta^a = \varepsilon^{ab}\theta_b, \qquad \theta_a = \varepsilon_{ab}\theta^b, \qquad \bar{\theta}^{\dot{a}} = \varepsilon^{\dot{b}}\bar{\theta}_{\dot{b}}, \qquad \bar{\theta}_{\dot{a}} = \varepsilon_{\dot{a}\dot{b}}\bar{\theta}^{\dot{b}},$$

where $\varepsilon^{12} = \varepsilon_{21} = \varepsilon^{\dot{1}\dot{2}} = \varepsilon_{\dot{2}\dot{1}} = 1$ and $\varepsilon^{ab} = -\varepsilon^{ba}$, $\varepsilon_{ab} = -\varepsilon_{ba}$, $\varepsilon^{\dot{a}\dot{b}} = -\varepsilon^{\dot{b}\dot{a}}$, $\varepsilon_{\dot{a}\dot{b}} = -\varepsilon_{\dot{b}\dot{a}}$.

The covariant derivative in the superspace is

$$D_a = \partial_a + i\sigma^\mu_{a\dot{a}}\bar{\theta}^{\dot{a}}\partial_\mu, \qquad \bar{D}_{\dot{a}} = -\bar{\partial}_{\dot{a}} - i\theta^a \sigma^\mu_{a\dot{a}}\partial_\mu.$$

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